NOTES

- NOTES WITH MIND MAPS -

MATHEMATICS

(ALGEBRAIC EXPRESSIONS)

7^{TH}

Algebraic Expressions

Introduction to Algebraic Expressions

Constant

Constant is a quantity which has a fixed value.

Terms of Expression

Parts of an expression which are formed separately first and then added are known as terms. They are added to form expressions.

Example: Terms 4x and 5 are added to form the expression (4x +5).

Coefficient of a term

The numerical factor of a term is called coefficient of the term.

Example: 10 is the coefficient of the term 10xy in the expression 10xy+4y.

Algebra as Patterns

Writing Number patterns and rules related to them

If a natural number is denoted by n, its successor is (n + 1).

Example: Successor of n=10 is n+1 =11.

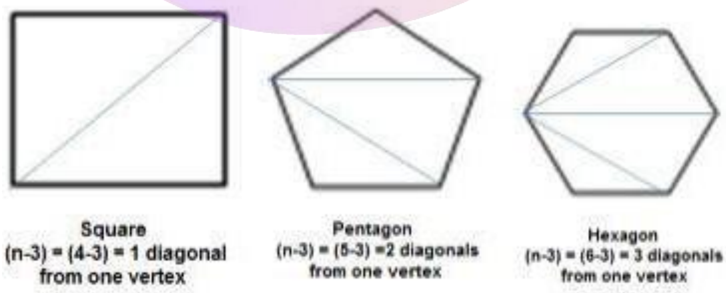
If a natural number is denoted by n, 2n is an even number and (2n+1) an odd number.

Example: If n=10, then 2n = 20 is an even number and 2n+1 = 21 is an odd number.

Writing Patterns in Geometry

Algebraic expressions are used in writing patterns followed by geometrical figures.

Example: Number of diagonals we can draw from one vertex of a polygon of n sides is (n - 3).



Definition of Variables

Any algebraic expression can have any number of variables and constants.

Variable

A variable is a quantity that is prone to change with the context of the situation.

a,x,p,… are used to denote variables.

Constant

- It is a quantity which has a fixed value.
- In the expression 5x + 4, the variable here is x and the constant is 4.
- The value 5x and 4 are also called terms of expression.
- In the term 5x, 5 is called the coefficient of x. Coefficients are any numerical factor of a term.

Factors of a term

Factors of a term are quantities which can not be further factorised. A term is a product of its factors.

Example: The term –3xy is a product of the factors –3, x and y.

Formation of Algebraic Expressions

- Variables and numbers are used to construct terms.
- These terms along with a combination of operators constitute an algebraic expression.
- The algebraic expression has a value that depends on the values of the variables.
- For example, let $6p^2 - 3p+5$ be an algebraic expression with variable p

  The value of the expression when p=2 is,

  $6(2)^2 - 3(2) + 5$

  $\Rightarrow 6(4) - 6 + 5 = 23$

  The value of the expression when p=1 is,

  $6(1)^2 - 3(1) + 5$

  $\Rightarrow 6 - 3 + 5 = 8$

Like and Unlike Terms

Like terms

Terms having same algebraic factors are like terms.

Example: 8xy and 3xy are like terms.

Unlike terms

Terms having different algebraic factors are unlike terms.

Example: 7xy and −3x are unlike terms.

Monomial, Binomial, Trinomial and Polynomial Terms

Types of expressions based on the number of terms

Based on the number of terms present, algebraic expressions are classified as:

- **Monomial:** An expression with only one term.

  Example: 7xy, −5m, etc.

- **Binomial:** An expression which contains two, unlike terms.

  Example: 5mn + 4, x + y, etc

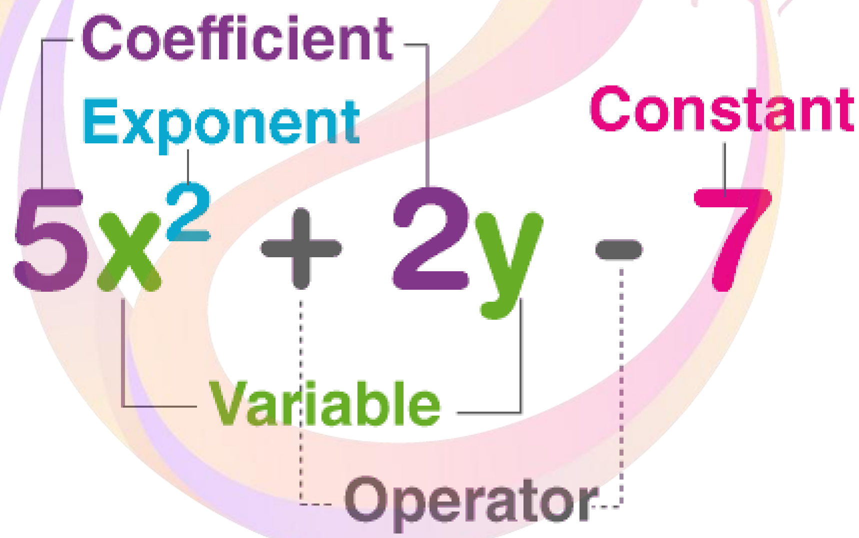
- **Trinomial:** An expression which contains three terms.

  Example: x + y + 5, a + b + ab, etc.

Polynomials

An expression with one or more terms.

Example: x + y, 3xy + 6 + y, etc.



Notation

The polynomial function is denoted by P(x) where x represents the variable. For example,

$P(x) = x^2 - 5x + 11$

If the variable is denoted by a, then the function will be P(a)

Degree of a Polynomial

The degree of a polynomial is defined as the highest degree of a monomial within a polynomial. Thus, a polynomial equation having one variable which has the largest exponent is called a degree of the polynomial.

Polynomial	Degree	Example
Constant or Zero Polynomial	0	6
Linear Polynomial	1	3x+1
Quadratic Polynomial	2	$4x^2+1x+1$
Cubic Polynomial	3	$6x^3+4x^3+3x+1$
Quartic Polynomial	4	$6x^4+3x^3+3x^2+2x+1$

Properties of polynomial

Some of the important properties of polynomials along with some important polynomial theorems are as follows:

Property 1: Division Algorithm

If a polynomial P(x) is divided by a polynomial G(x) result in quotient Q(x) with remainder R(x), then,

$P(x) = G(x) \bullet Q(x) + R(x)$

Property 2: Bezout's Theorem

Polynomial P(x) is divisible by binomial (x – a) if and only if P(a) = 0.

Property 3: Remainder Theorem

If P(x) is divided by (x – a) with remainder r, then P(a) = r.

Property 4: Factor Theorem

A polynomial P(x) divided by Q(x) results in R(x) with zero remainders if and only if Q(x) is a factor of P(x).

Property 5: Intermediate Value Theorem

If P(x) is a polynomial, and $P(x) \neq P(y)$ for $(x < y)$, then P(x) takes every value from P(x) to P(y) in the closed interval [x, y].

Property 6

The addition, subtraction and multiplication of polynomials P and Q result in a polynomial where,

Degree $(P \pm Q) \leq$ Degree (P or Q)

Degree $(P \times Q)$ = Degree (P) + Degree(Q)

Property 7

If a polynomial P is divisible by a polynomial Q, then every zero of Q is also a zero of P.

Property 8

If a polynomial P is divisible by two coprime polynomials Q and R, then it is divisible by (Q • R).

Property 9

If $P(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$ is a polynomial such that $\deg(P) = n \geq 0$ then, P has at most "n" distinct roots.

Property 10: Descartes' Rule of Sign

The number of positive real zeroes in a polynomial function P(x) is the same or less than by an even number as the number of changes in the sign of the coefficients. So, if there are "K" sign changes, the number of roots will be "k" or "(k – a)", where "a" is some even number.

Property 11: Fundamental Theorem of Algebra

Every non-constant single-variable polynomial with complex coefficients has at least one complex root.

Property 12

If P(x) is a polynomial with real coefficients and has one complex zero (x = a – bi), then x = a + bi will also be a zero of P(x). Also, $x^2 - 2ax + a^2 + b^2$ will be a factor of P(x).

Polynomial Functions

A polynomial function is an expression constructed with one or more terms of variables with constant exponents. If there are real numbers denoted by a, then function with one variable and of degree n can be written as:

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-2}x^2 + a_{n-1}x + a_n$$

Addition and Subtraction of Algebraic Equations

- Mathematical operations like addition and subtraction can be applied to algebraic terms.
- For adding or subtracting two or more algebraic expression, like terms of both the expressions are grouped together and unlike terms are retained as it is.
- Sum of two or more like terms is a like term with a numerical coefficient equal to the sum of the numerical coefficients of all like terms.
- Difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.
- For example, 2y + 3x − 2x + 4y

$\Rightarrow x\ (3 - 2) + y\ (2 + 4)$

$\Rightarrow x + 6y$

- Summation of algebraic expressions can be done in two ways:

  Consider the summation of the algebraic expressions $5a^2 + 7a + 2ab$ and $7a^2 + 9a + 11b$

- **Horizontal method**

  Consider three algebraic expressions $5xy - 3x^2 - 12y + 5x$, $xy - 3x - 12yz + 5x^3$ and $y - 6x^2 - zy + 5x^3$.

  Step 1: Write the given algebraic expressions using an additional symbol.

  $(5xy - 3x^2 - 12y + 5x) + (xy - 3x - 12yz + 5x^3) + (y - 6x^2 - zy + 5x^3)$

  Step 2: Open the brackets and multiply the signs.

  $5xy - 3x^2 - 12y + 5x + xy - 3x - 12yz + 5x^3 + y - 6x^2 - zy + 5x^3$

  Step 3: Now, combine the like terms.

  $(5xy + xy) + (-3x^2 - 6x^2) + (-12y + y) + (5x - 3x) + (-12yz - yz) + (5x^3 + 5x^3)$

  Step 4: Add the coefficients. Keep the variables and exponents on the variables the same.

  $6xy - 9x^2 - 11y + 2x - 13yz + 10x^3$

  Column Method

  In this method, the given expression must be written column wise one below the other. For adding two or more algebraic expressions the like terms of both the expressions are grouped together. The coefficients of like terms are added together using simple addition techniques and the variable which is common is retained as it is. The, unlike terms, are retained as it is and the result obtained is the addition of two or more algebraic expressions.

$+5xy$	$-3x^2$	$-12y$	$+5x$	$+0yz$	$+0x^3$
$+xy$	$+0x^2$	$+0y$	$-3x$	$-12yz$	$+5x^3$
$+0xy$	$-6x^2$	$+1y$	$+0x$	$-yz$	$+5x^3$
$6xy$	$-9x^2$	$-11y$	$+2x$	$-13yz$	$+10x^3$

- **Vertical method**

  $5a^2 + 7a + 2ab$

$7a^2 + 9a + 11b$

$12a^2+16a+2ab+11b$

The knowledge of like and unlike terms is crucial while studying addition and subtraction of algebraic expressions because the operation of addition and subtraction can only be performed on like terms. The terms whose variables and their exponents are the same are known as like terms and the terms having different variables are unlike terms.

Example: $-5x^2 + 12\ xy - 3y + 7x^2 + xy$

In the given algebraic expression, $-5x^2$ and $7x^2$ are like since both the terms have x^2 as the common variable. Similarly, 12xy and xy are like terms.

Use of algebraic expressions in the formula of the perimeter of figures

Algebraic expressions can be used in formulating perimeter of figures.

Example: Let l be the length of one side then the perimeter of:

Equilateral triangle = 3l

Square = 4l

Regular pentagon = 5l

Use of algebraic expressions in formula of area of figures

Algebraic expressions can be used in formulation area of figures.

Example: Let l be the length and b be the breadth then the area of:

Square = l2

Rectangle = l × b = lb

Triangle = $\frac{b \times h}{2}$, where b and h are base and height, respectively.